Assessment of learning goals in the
PhD Program in Mathematics
Academic year 2016/17

Summary of Learning goals for the PhD program:

1. Students learn a substantial body of mathematics in introductory and research level graduate courses in mathematics.
2. Students complete a dissertation under the guidance of an advisor. The dissertation should make an original and substantive contribution to its subject matter.
3. Students demonstrate breadth within the learning experiences.
4. Students present research in seminar talks, conferences or publications.
5. Students communicate complex ideas in a clear and understandable manner.

Assessment activities: The above learning goals were assessed by the following activities:

- (Learning Goal 1) Problems and outcome on a midterm in Math 721 were analyzed and the results were submitted for discussion to the Graduate Program Committee.
- (Learning goal 1) The outcomes of the bi-annual Ph.D. qualifying exams were presented to the Graduate Program Committee for discussion.
- (Learning Goal 2) The outcomes of dissertation research were assessed by dissertation committees. The department keeps historical data about the topics of the dissertation research. The department also gathers records about academic and non-academic job placements following the graduation.
- (Learning Goal 3) Breadth requirements are monitored by the Graduate Coordinator and the Director of Graduate Studies.
- (Learning goals 4-5) Students after the second year in the program provided an annual report on their research activities. Graduate advisors provided a short annual report on students with dissertator status. These reports were reviewed by the Director of Graduate Studies. In some cases the DGS initiated a conversation with the advisor or the student (or both). Feedback using a rating was provided to all students which are beyond the period of guaranteed support.

Recommendations: The Graduate Program Committee made no specific recommendations.

Assessment of learning goals in the Masters Program Mathematics: Foundations of Advanced Mathematics
Academic year 2016/17

Summary of Learning goals for the MA-FAS program:

1. Students learn a substantial body of mathematics presented in introductory graduate level courses in mathematics.
2. Students select and utilize appropriate methodologies to solve problems.
3. Students communicate clearly in written presentations.
4. Students recognize and apply principles of ethical and professional conduct.

Assessment activities:

- (Learning Goal 1-2-3) Problems and outcome on a midterm in Math 721 were analyzed and the results were submitted for discussion to the Graduate Program Committee.
- (Learning goal 1-2-3) Some students in the program took part in the bi-annual Ph.D. qualifying exams. Outcomes of the exams were presented to the Graduate Program Committee for discussion.
- (Learning goal 4) Faculty were asked to communicate any problematic incidents to the Director of Graduate Studies.

Recommendations: The Graduate Program Committee made no specific recommendations.

Assessment of a Midterm exam in Mathematics 721

The midterm exam was given in class (50 min.), in week 7. The exam covered up to chapter 2.4 in the textbook by Folland. The students were asked to work on four problems, with the recommendation that they solve the first three problems, and work on the fourth if time permits. The problems were essentially taken out of a collection of either assigned or recommended exercises. This was announced to the students ahead of the exam.

The problems were significantly easier than what Ph.D. students should expect on the Analysis Qualifying Exam. This was communicated to the students after returning the exam.

Problem 1. (i) Let $(X, \mathcal{M}, \mu)$ be a measure space. For $k \in \mathbb{N}$ let E_k be measurable sets, and assume that $\sum_{k=1}^{\infty} \mu(E_k) < \infty$. Show that the set

$$E := \{x \in X : x \in E_k \text{ for infinitely many } k\}$$

has measure zero.

(ii) Find a sequence of measurable sets $E_k \subset [0,1]$ such that the Lebesgue measure of E_k is at most $1/k$ and such that the set E defined in (i) has positive Lebesgue measure.

Comments: Many students solved part (i), the standard Borel Cantelli lemma, correctly, with the usual proof.

Some students confused $\limsup E_k$ *and* $\liminf E_k$. *Some got the correct definition as* $\limsup E_k$ *but failed to use that the tails of an absolutely convergent series tend to* 0.

Many students understood the obvious idea needed for (ii) but only some could formulate an accurate proof with correct notation.

Problem 2. Let m be Lebesgue measure on $I = [0,1]$, and let $f \in L^1(I)$. Define

$$A(x) = \int_{[0,x]} f dm$$

and show that A is continuous on I.

Comments: Many students got this essentially right. Students like to apply the dominated convergence theorem. Some students failed to mention basic estimates such as $|\int_E f dm| \le \int_E |f| dm$.

Problem 3. State Egorov's theorem. Use it to prove the following theorem by Lusin:

Let $f : [0,1] \to \mathbb{R}$ be a Lebesgue measurable function. Then for every $\varepsilon > 0$ there is a set E_ε of Lebesgue measure at most ε so that the restriction of f to $[0,1] \setminus E_\varepsilon$ is continuous.

Comments: Most students were able to state Egorov's theorem (as the importance of this theorem was pointed out in class several times). Lusin's theorem is an exercise in Folland but was actually explained in class. So students saw before the idea of approximating by continuous functions, and using this in conjunction with Egorov's theorem. Several students showed the result only for L^1 *functions.*

Problem 4. Let $(X, \mathcal{M}, \mu)$ be a measure space. Suppose that $\{f_n\}_{n=1}^\infty$ is a sequence of measurable functions such that f_n converges to f in measure and such that $|f_n(x)| \le g(x)$ a.e. for a function $g \in L^1$.

Prove that

$$\lim_{n\to\infty} \int |f - f_n| d\mu = 0\,.$$

Comments: It was pointed out on the exam sheet that this is somewhat easier if one assumes $\mu(X) < \infty$*. Students were given the discretion to prove the result under this additional assumption. A very similar problem was assigned before (and some students used the assigned problem to give a very short solution).*

Many students used the fact that a sequence in a metric space converges iff every subsequence has a further subsequence which converges. This fact can be used in conjunction with the standard dominated convergence theorem to solve the problem. A more quantitative approach (not relying on the dominated convergence theorem) is preferable but was only taken by a few students.